MIXED BOUNDARY VALUE PROBLEM ON HYPERSURFACES

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Abstract. *The purpose of the present paper is to investigate the mixed Dirichlet-Neumann boundary value problems for the anisotropic Laplace-Beltrami equation* $\mathrm{div}_{\mathscr{C}}(A\nabla_{\mathscr{C}}\varphi) = f$ *on a smooth hypersurface* $\mathscr{C}$ *with the boundary* $\Gamma = \partial\mathscr{C}$ *in* $\mathbb{R}^n$. $A(x)$ *is an* $n \times n$ *bounded measurable positive definite matrix function. The boundary is decomposed into two non-intersecting connected parts* $\Gamma = \Gamma_D \cup \Gamma_N$ *and on the part* Γ_D *the Dirichlet boundary conditions while on* Γ_N *the Neumann boundary condition are prescribed. The unique solvability of the mixed BVP is proved, based upon the Green formulae and Lax-Milgram Lemma.*

We also prove the invertibility of the perturbed operator in the Bessel potential spaces $\mathrm{div}_{\mathscr{S}}(A\nabla_{\mathscr{S}})+\mathscr{H}I \ : \ \mathbb{H}^s_p(\mathscr{S}) \to \mathbb{H}^{s-2}_p(\mathscr{S})$ *for a smooth hypersurface* $\mathscr{S}$ *without boundary for arbitrary* $1 < p < \infty$ *and* $-\infty < s < \infty$, *provided* $\mathscr{H}$ *is smooth function, has non-negative real part* $\mathrm{Re}\,\mathscr{H}(t) \geqslant 0$ *for all* $t \in \mathscr{S}$ *and* $\mathrm{mes}\,\mathrm{supp}\,\mathrm{Re}\,\mathscr{H} \neq 0$. *Further the existence of the fundamental solution to* $\mathrm{div}_{\mathscr{S}}(A\nabla_{\mathscr{S}})$ *is proved, which is interpreted as the invertibility of this operator in the setting* $\mathbb{H}^s_{p,\#}(\mathscr{S}) \to \mathbb{H}^{s-2}_{p,\#}(\mathscr{S})$, *where* $\mathbb{H}^s_{p,\#}(\mathscr{S})$ *is a subspace of the Bessel potential space and consists of functions with mean value zero.*